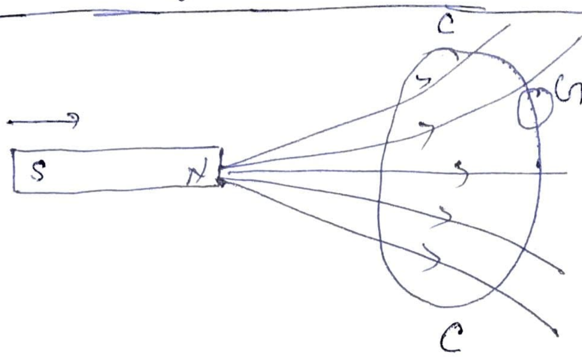
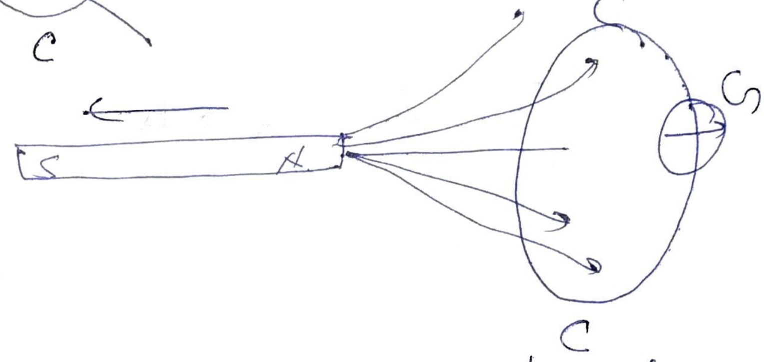
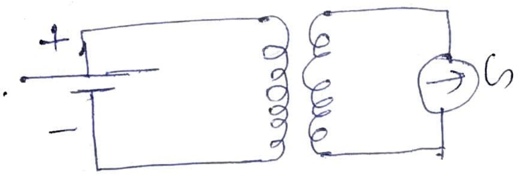
Electro-magnetic Induction - Faraday's Law







Induced electric field is produced due to change in magnetic flux. The magnitude of induced e.m.f is proportional to the rate of change of magnetic flux.

$$|\varepsilon| \propto \frac{d\phi}{dt} \quad \text{——(1)}$$

$|\varepsilon| \rightarrow$ e.m.f, $\phi \rightarrow$ magnetic flux

Direction of induced e.m.f is given by Lenz's law

The direction of e.m.f is such that the magnetic flux produced to the current is such that it opposes the cause of production of current.

eq^n (1)

$$\varepsilon = -\frac{d\phi}{dt} \quad \text{——(2)}$$

Integral and differential form of Faraday's Law

The induced e.m.f in a coil

$$\varepsilon = \int_l \vec{E}\cdot\hat{l}\, dl$$

and the magnetic flux passing through the coil

$$\phi = \int_s \vec{B} \cdot \vec{ds}$$

But $\quad \varepsilon = (-) \frac{d\phi}{dt}$

$$\Rightarrow \quad \varepsilon = \int_l \vec{E} \cdot \hat{l}\, dl = (-) \frac{d}{dt} \int \vec{B} \cdot \vec{ds}$$

The shape and position of s does not change with time, so

$$\varepsilon = \int_l \vec{E} \cdot \hat{l}\, dl = (-) \int_s \frac{d\vec{B}}{dt} \cdot ds \quad \text{---(3)}$$

Prob: There are 1000 turns per meter in a long solenoid of cross section $4\,cm^2$. The current changes at the rate $100\,A/s$ in these turns. Find induced emf in the coil.

Soln: Magnetic field along the axis of current carrying solenoid

$$B = \mu_0 n I$$

flux

$$\phi = BA = \mu_0 N I A \quad \text{---(i)}$$

$$\varepsilon = \frac{d\phi}{dt} = \frac{d}{dt}\left[\mu_0 n I A\right]$$

$$= \mu_0 n A \frac{dI}{dt}$$

$$= 4\pi \times 10^{-7} \times 1000 \times \frac{4}{100 \times 100} \times 100$$

$$= 16\pi \times 10^{-6}$$

$$= 16\pi\,\mu V$$

Answer

Integral form of Faraday's law

$$e = \int_l E.\hat{l}\, dl = (-)\int_s \frac{d\vec{B}}{dt}.d\vec{s} \quad \text{---} (1)$$

Stokes theorem

$$\int \vec{E}\cdot\hat{l}\, dl = \int_s \nabla\times\vec{E}\cdot ds$$

$$\Rightarrow \int_s (\nabla\times E)\cdot ds = -\int \frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}$$

or $$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{---} (2)$$

Differential form of Faraday's law

eq^n (2) and Gauss law $\nabla.\vec{E} = \rho/\varepsilon_0$ show that electric field has non-conservative part due to changing magnetic flux density & has conservative part due to electric charge density.

These are two kind of sources for electromagnetic fields.

first kind of sources → Energy is conserved in cyclic process

Such kind of conservative or irrotational system are presented by

$$\nabla.E = \rho/\varepsilon_0 \text{ in electrostatics}$$

$$\text{where } E = -\nabla\phi$$

No curl exist for such kind of sources because

$$\nabla\times\vec{E} = \nabla\times\nabla\phi = 0$$

Second kind of Sources → related to such systems in which energy is transferred in cyclic process such as magnetic field of solenoid. Such field are represented by curl → divergence does not exist.

Any vector field can be represented uniquely if its divergence and curl sources exist.

↓ Helmholtz Theorem

$$\nabla \cdot \nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \cdot \vec{B}) = 0$$

$$\Rightarrow \nabla \cdot \vec{B} = 0 \qquad B \rightarrow \text{is solenoidal}$$

Self Inductance and Mutual Inductance

The magnetic flux of a current carrying coil is given by

$$\phi \propto I$$

or

$$\phi = L I \quad \text{--- (1)}$$

↓

Self Inductance of coil

If coil is static but magnetic field or flux changes with time. If current is time dependent flux will also be time dependent $\phi = \phi(t)$

$$\Rightarrow \frac{d\phi}{dt} = \frac{d\phi}{dI}\frac{dI}{dt} = L\frac{dI}{dt} \quad \text{--- (2)}$$

here $\frac{d\phi}{dI} = L$

$$\Rightarrow \varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \quad \text{--- (3)}$$

↓

depends on geometry of coil.

Self inductance of a wire in the form of a solenoid will be more than the self inductance of a straight wire.

$L \rightarrow$ also depends on the medium in which coil is placed

If unit current passes through the coil the magnetic flux passing through coil is equal to its self inductance

$$\phi = L \quad \text{for } I = 1$$

unit of self inductance is <u>1 Henry.</u>

Self inductance of a circuit will be unit if the current in it changes 1 amp/sec and produces 1 volt e.m.f.

If I_1 current is passed through a coil. If a secondary coil is brought near, magnetic flux is produced in second coil ϕ_2

$$\phi_2 = L_{21} I_1 \quad \text{---(4)}$$

Similarly magnetic flux ϕ_1 related to current in second circuit I_2

$$\phi_1 = L_{12} I_2 \quad \text{---(5)}$$

Potential energy of system in first condition

$$U_p = -\phi_2 I_2$$

$$= -L_{21} I_1 I_2 \quad \text{---(6)}$$

Similarly potential energy in second condition

$$U_p = -\phi_1 I_1 = -L_{12} I_2 I_1 \quad \text{---(7)}$$

From (6) and (7)

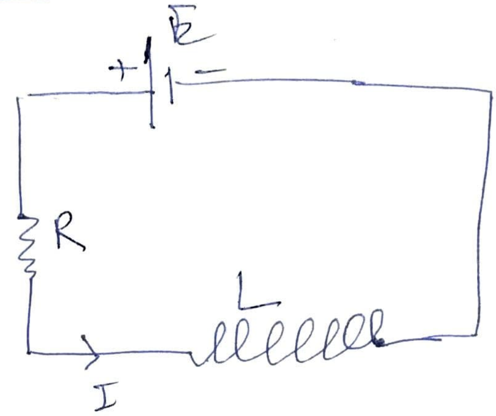
$$L_{21} = L_{12} = M$$

$\downarrow$

<u>Mutual Inductance between two circuits</u>

Energy in Magnetic field

Energy in a field is equal to the work done to establish it.



Magnetic energy stored in an inductor

Let the current in circuit at time t is $I(t)$. The voltage drop across coil is $L\frac{dI}{dt}$.

$\Rightarrow$ e.m.f $\quad \varepsilon = -L\frac{dI}{dt}$

From Ohm Law

$$\varepsilon - L\frac{dI}{dt} = RI \quad \text{---(7)}$$

Work done by e.m.f ε in moving dq charge

$$dW = \varepsilon\, dq = \varepsilon I\, dt$$

$$\Rightarrow \frac{dW}{dt} = \varepsilon I \quad \text{---(8)}$$

$$\Rightarrow \frac{dW}{dt} = \varepsilon I = LI\frac{dI}{dt} + RI^2$$

$$\Rightarrow W = \int_0^T \varepsilon I\, dt = L\int_0^T I\frac{dI}{dt}\,dt + R\int_0^T I^2\, dt$$

$$= \underbrace{\frac{1}{2}LI_T^2}_{\text{energy stored in inductor}} + \underbrace{R\int_0^T I^2\, dt}_{\text{energy dissipation in ressistance } R} \quad \text{---(9)}$$